Sample Placement Exam for Calculus Readiness

Work the following problems without the assistance of a calculator. Once you have completed the entire problem set check your answers. There are 25 problems here; a score less than 20 indicates you are not ready for calculus.

1) Find the real solutions to each:
   a) $2x = 8 - x^2$
   d) $\frac{x^3 - 2x}{x^3 + 3x + 1} = 0$
   g) $e^{3x} = 100$
   b) $3x = 8 - x^2$
   e) $x + \sqrt{x} - 6 = 0$
   h) $\log_2 1 + \log_2 (t + 1) = 1$
   c) $\frac{3}{x} - \frac{3}{x + 2} = 2$
   f) $\ln x - \ln 3 = 2$
   i) $2 \cos x - \sqrt{2} = 0$, $0 \leq x \leq 2\pi$

2) Determine the exact coordinates of the point(s) where the following line(s) and curve(s) intersect.
   a) $3x + 4y = 5$ and $x - 2y = -6$
   b) $y = 8 - x^2$ and $y = 7x$

3) Find the equation of the line:
   a) through $(2, 7)$ and $(-5, 1)$
   b) parallel to $3x + 4y = 5$ and through $(2, 7)$

4) Solve the following inequalities and write the solution using interval notation.
   a) $x - 5 \leq \frac{1}{2} + 3x$
   b) $-3 \leq 1 - 2x < 4$

5) Express $\frac{1}{u} - \frac{3}{v}$ as a single fraction.

6) Express $\frac{\sin \theta}{1 + \cos \theta} + \frac{\cos \theta}{\sin \theta}$ as a single trigonometric function.

7) Evaluate.
   a) $\left(\frac{8x^{11}y^{-3}}{y^6z^3}\right)^{\frac{4}{3}}$
   b) $\tan\left(\frac{\pi}{3}\right)$
   c) $\sin^{-1}\left(-\frac{1}{2}\right)$

8) Let $f(x) = 2x^2 - 2x$.
   a) Compute $f(x + h)$
   b) Simplify $\frac{f(x + h) - f(x)}{h}$

9) A 10 foot ladder is leaning against a vertical building. Let $x$ be the distance along the floor from the wall to one end of the ladder and let $y$ be the vertical distance from the floor to where the top of the ladder touches the wall. Write an equation expressing $y$ in terms of $x$.

10) The cost $C$ of building a highway through a certain section of the country is proportional to its length $L$. A 2.5 mile section costs $1,000,000$.
   a) Express the cost as a function of length.
   b) Compute the cost of building 15 miles of highway.
Calculus Readiness Test Answers

1) a) $x = -4, x = 2$  
   b) $x = -\frac{3}{2} \pm \frac{\sqrt{41}}{2}$  
   c) $x = -3, x = 1$  
   d) $x = 0, x = 2$  
   e) $x = 4$  
   f) $x = 3e^2$  
   g) $t = \frac{1}{3} \ln(100)$  
   h) $t = 1$  
   i) $x = \frac{\pi}{4}, x = \frac{7\pi}{4}$

2) a) $\left(-\frac{7}{5}, -\frac{23}{10}\right)$  
   b) $(-8, -56)$ and $(1, 7)$

3) a) $y = \frac{6}{7}x - \frac{37}{7}$  
   b) $y = -\frac{3}{4}x + \frac{17}{2}$

4) a) $[-11, \infty)$  
   b) $\left[-\frac{3}{2}, 2\right]$  

5) $\frac{v - 3u}{nv}$

6) $\csc \theta$

7) a) $y^{12}z^4$  
   b) $\sqrt{3}$  
   c) $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$

8) a) $2x^2 + 4xh + 2h^2 - 2x - 2h$  
   b) $4x + 2h - 2$

9) $y = \sqrt{100 - x^2}$

10) a) $C = 400,000$L  
    b) $\$6,000,000$